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Rayleigh instability in Hall thrusters

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Gradient-driven Rayleigh-type instabilities in Hall plasma thrusters are analyzed using linearized two-fluid hydrodynamic equations. Necessary instability conditions and a general criterion for stability of azimuthally propagating perturbations are derived. For a simplified model of the axial distribution of parameters inside the thruster channel, the growth rate of an unstable wave, resonant with the azimuthal electron flow, is obtained. The frequency and phase relations are related to the results of experimental investigations of high-frequency oscillations in Hall thrusters. © *2004 American Institute of Physics.* [DOI: 10.1063/1.1647565]

I. INTRODUCTION

The importance of plasma oscillations for the successful operation of Hall current plasma thrusters has been long recognized (see, e.g., Ref. 1). These oscillations play an important role in controlling the transport, conduction, and mobility in these devices, thus directly affecting their performance. These oscillations are also important in matching the thruster to the power processing circuit.

The presence of plasma density and magnetic field gradients is one of the main sources for plasma instabilities.[2,3] However, despite general agreement with the early experimental data,[4] these models do not explain all of the observations, such as, for example, the presence of high-frequency (MHz range) plasma oscillations, which were recently detected and characterized.[5,6]

In this paper we study two-dimensional plasma perturbations in a Hall current plasma thruster on the basis of two-fluid hydrodynamic theory. We focus on modes with purely azimuthal propagation, suggested by the experimental findings.[5] These findings include plasma oscillations in the presence of sharp gradients of plasma parameters, typical for operating regimes of state-of-the art Hall thrusters.[7] We also include collisional terms for electrons. We show that Rayleigh-type instability of azimuthal electrostatic waves appears. We determine the instability frequency and growth rate for a particular model of steady-state axial distribution of parameters inside the thruster channel.

II. THE MODEL

Consider a Hall thruster with a cold two-component plasma, consisting of ions and electrons immersed in the magnetic field B_o, such that, on the scale of the device, electrons are magnetized, while ions are unmagnetized,

$$\rho_e \ll L \ll \rho_i . \tag{1}$$

We treat the annular channel of the thruster as flat and neglect the channel curvature, as well as the axial component of magnetic field and the changes of any variables in the radial direction, thus simplifying the problem to two-dimentional geometry. Limiting our study to quasielectrostatic waves, we have

$$\vec{E} = -\nabla \phi \tag{2}$$

both for zeroth order and perturbed values of electric field and plasma potential.

The ion motion is governed by fluid equations,

$$\frac{\partial N_i}{\partial t} + \nabla(\vec{v}_i N_i) = 0, \tag{3}$$

$$\frac{\partial \vec{v}_i}{\partial t} + (\vec{v}_i \cdot \nabla)\vec{v}_i = \frac{e}{M}\vec{E}. \tag{4}$$

The zeroth order solution is the axial flow of unmagnetized ions being accelerated by the electric field in the channel according to

$$v_o \frac{dv_o}{dx} = \frac{e}{M} E_o . \tag{5}$$

The linearized system for the small perturbations of ion density and velocity is then written as follows:

$$\frac{\partial n_i}{\partial t} + \vec{v}_o \frac{\partial n_i}{\partial x} + n_o \nabla \cdot \vec{v}_i = 0, \tag{6}$$

$$\frac{\partial \vec{v}_i}{\partial t} + v_o \frac{\partial \vec{v}_i}{\partial x} = \frac{e}{M}\vec{E}_1 . \tag{7}$$

We consider perturbations of all variables of the form

$$A(\vec{r},t) = A(x)\exp(i\omega t - ik_y y).$$

For the ion motion we can also use the assumption that the frequency of the perturbations we consider is high enough compared to the ion flow velocity $v_0 \ll \omega/L$, where L is the inhomogeneity length along the channel. Then we obtain the following expression for the perturbed ion density in terms of the perturbed electric field

$$n_i = \frac{e n_0}{M} \frac{-\dfrac{\partial E_x}{\partial x} + i k_y E_y}{\omega^2}, \tag{8}$$

or in terms of potential perturbation according to (2),

$$n_i=\frac{en_0}{M}\frac{1}{\omega^2}\left(\frac{\partial^2\phi}{\partial x^2}-k_y^2\phi\right). \tag{9}$$

The electron motion is governed by the similar set of continuity and momentum equations,

$$\frac{\partial N_e}{\partial t}+\nabla(\vec{v}_e N_e)=0, \tag{10}$$

$$\frac{\partial\vec{v}_e}{\partial t}+(\vec{v}_e\cdot\nabla)\vec{v}_e=\frac{e}{m}\left(\vec{E}+\frac{1}{c}\vec{v}_e\times\vec{B}\right)-\nu_e\vec{v}_e. \tag{11}$$

In the zeroth order, assuming $\nu_e \ll \Omega_e$ the electrons move in the $\hat{y}$ direction with the drift velocity

$$u_o=-c\frac{E_o}{B_o}. \tag{12}$$

The linearized system for the small perturbations of electron density and velocity is written as follows:

$$\frac{\partial n_e}{\partial t}+(\vec{u}_o\cdot\nabla)n_e+n_o(\nabla\cdot\vec{v}_e)+v_{ex}\frac{\partial n_o}{\partial x}=0, \tag{13}$$

$$\frac{\partial\vec{v}_e}{\partial t}+(\vec{u}_o\cdot\nabla)\vec{v}_e+(\vec{v}_e\cdot\nabla)\vec{v}_0 =-\frac{e}{M}\left(\vec{E}_1+\frac{1}{c}\vec{v}_e\times\vec{B}_o\right)-\nu_e\vec{v}_e. \tag{14}$$

The momentum equation (14) yields the following system for the $\hat{x}$ and $\hat{y}$ components of the oscillating electron velocity perturbation:

$$i(\omega-k_yu_0-i\nu_e)v_x=\frac{e}{m}\frac{\partial\phi}{\partial x}-\Omega_e v_y, \tag{15}$$

$$i(\omega-k_yu_0-i\nu_e)v_y+v_x\frac{\partial u_0}{\partial x}=-ik_y\frac{e}{m}\phi+\Omega_e v_x. \tag{16}$$

Here we have introduced the electron gyrofrequency $\Omega_e=eB_0/mc$. We resolve this system to obtain the following expressions for v_x and v_y:

$$v_x=\frac{e}{m}\,\frac{i(\omega-k_yu_0-i\nu_e)\dfrac{\partial\phi}{\partial x}+ik_y\Omega_e\phi}{\Omega_e^2-(\omega-k_yu_0-i\nu_e)^2-\Omega_e\dfrac{\partial u_0}{\partial x}}, \tag{17}$$

$$v_y=\frac{1}{\Omega_e}\frac{e}{m}\frac{\partial\phi}{\partial x}+\frac{e}{m}\times\frac{(\omega-k_yu_0-i\nu_e)^2\dfrac{\partial\phi}{\partial x}-i(\omega-k_yu_0-i\nu_e)\Omega_e ik_y\phi}{\left(\Omega_e^2-(\omega-k_yu_0-i\nu_e)^2-\Omega_e\dfrac{\partial u_0}{\partial x}\right)\Omega_e}. \tag{18}$$

For typical Hall thruster operating conditions to satisfy the condition (1), the applied magnetic field is of the order of 10^2 G. The electron gyrofrequency in such a case is of the order of a few GHz and is substantially larger than frequencies of the oscillations we consider in our model,

$$\Omega_e \gg \omega,\ \ k_yu_0,\ \ \partial u_0/\partial x,\ \ \nu_e. \tag{19}$$

This condition allows the expression for electron velocities to be simplified,

$$v_x=\frac{e}{m}\frac{1}{\Omega_e}\left[\frac{i\hat{\omega}}{\Omega_e}\frac{\partial\phi}{\partial x}+ik_y\phi+\frac{1}{\Omega_e}\frac{\partial u_0}{\partial x}ik_y\phi\right], \tag{20}$$

$$v_y=\frac{e}{m}\frac{1}{\Omega_e}\left[\frac{\partial\phi}{\partial x}-\frac{i\hat{\omega}}{\Omega_e}ik_y\phi+\frac{i\hat{\omega}}{\Omega_e^2}\frac{\partial u_0}{\partial x}ik_y\phi\right], \tag{21}$$

where $\hat{\omega}=(\omega-k_yu_0-i\nu_e)$. We can now substitute this into the electron continuity equation, rewritten for harmonic electrostatic perturbations as

$$n_e=\frac{n_0}{i(\omega-k_yu_0)}\left[ik_yv_y-\frac{\partial v_x}{\partial x}+v_x\frac{1}{n_0}\frac{\partial n_0}{\partial x}\right]. \tag{22}$$

After some vigorous algebra, eliminating terms of higher order than $O(\omega/\Omega_e)$ we obtain the following expression for the electron density in terms of potential perturbation:

$$n_e=\frac{en_0}{m\Omega_e^2}\left(\left(k_y^2\phi-\frac{\partial^2\phi}{\partial x^2}\right)\left(1-\frac{i\nu_e}{\omega-k_yu_0}\right)+\frac{k_y}{\omega-k_yu_0}\left(\Omega_e\frac{\partial}{\partial x}\ln\frac{B_0}{n_0}-\frac{\partial^2u_0}{\partial x^2}\right)\phi\right). \tag{23}$$

Now we can substitute the obtained expressions for ion and electron density perturbations (9) and (23) into Poisson's equation

$$\nabla^2\phi=4\pi e(n_i-n_e), \tag{24}$$

which will yield the following equation for the perturbation of the plasma potential:

$$\left[\frac{\partial^2\phi}{\partial x^2}-k_y^2\phi\right]\left[1-\frac{\omega_{pi}^2}{\omega^2}+\frac{\omega_{pe}^2}{\Omega_e^2}\right]-\frac{\omega_{pe}^2}{\Omega_e^2}\frac{k_y\phi}{\omega-k_yu_0}\left(\Omega_e\frac{\partial}{\partial x}\ln\frac{B_0}{n_0}-\frac{\partial^2u_0}{\partial x^2}\right)-\frac{\omega_{pe}^2}{\Omega_e^2}\frac{i\nu_e}{\omega-k_yu_0}\left[\frac{\partial^2\phi}{\partial x^2}-k_y^2\phi\right]=0. \tag{25}$$

If we make yet another assumption about the frequency range of the oscillations, namely we consider that the frequency of these oscillations is much greater than the lower hybrid frequency,

$$\omega \gg \omega_{\mathrm{LH}}=\left(\frac{\omega_{pi}^2\Omega_e^2}{\omega_{pi}^2+\Omega_e^2}\right)^{1/2}, \tag{26}$$

then the equation (25) is further simplified to

$$\frac{\partial^2\phi}{\partial x^2}-k_y^2\phi-\frac{k_y\phi}{\omega-k_yu_0}\left(\Omega_e\frac{\partial}{\partial x}\ln\frac{B_0}{n_0}-\frac{\partial^2u_0}{\partial x^2}\right)-\frac{i\nu_e}{\omega-k_yu_0}\left[\frac{\partial^2\phi}{\partial x^2}-k_y^2\phi\right]=0. \tag{27}$$

III. RAYLEIGH INSTABILITY

In appropriate limits, Eq. (27) reduces to some well-known equations. First, consider the collisionless case, when Eq. (27) simplifies to

$$\frac{\partial^2\phi}{\partial x^2}-k_y^2\phi-\frac{k_y\phi}{\omega-k_yu_0}\left(\Omega_e\frac{\partial}{\partial x}\ln\frac{B_0}{n_0}-\frac{\partial^2u_0}{\partial x^2}\right)=0. \quad (28)$$

This equation is similar to the Rayleigh equation, well known in the fluid dynamics:[11]

$$\frac{\partial^2\Phi}{\partial x^2}-k_y^2\Phi+\frac{k_y\Phi}{\omega-k_yV_y}\left(\frac{\partial^2V_y}{\partial x^2}\right)=0, \quad (29)$$

where V_y is the flow velocity in the $\hat{y}$ direction and Φ is the so-called flow function, $V=\nabla\Phi$. It can be further analyzed using the methodology, presented in Ref. 12.

Following,[12] we treat (28) as an equation for the axial profile of electric potential perturbation $\phi(x)$, with ω and k_y being the free parameters. In slab geometry, perturbations can evolve both in space and in time. However, in the azimuthally symmetric channel of a real Hall thruster, the azimuthally propagating perturbation has to be periodic in the direction of propagation. Therefore, k_y can be only real, while ω can be complex, with its imaginary part representing the growth rate of the unstable oscillations. Also, due to the periodic boundary conditions in the $\hat{y}$ direction, possible values of k_y must be limited to a discrete set.

The boundary conditions for $\phi(x)$ can be chosen as

$$\phi(x_1)=0, \quad \phi(x_2)=0, \quad (30)$$

where x_1 and x_2 represent the boundaries of the investigated region. These conditions correspond to a firmly fixed value of potential at the anode of the thruster and at the virtual cathode, i.e., magnetic surface going through the cathode neutralizer. We therefore limit our case to perturbations, localized in the axial direction.

Using the Rayleigh theorem,[13] it can be shown that Im $\omega=0$, i.e., there will be no unstable oscillation, unless the following condition is met at some point $x=x_0$:

$$\left(\Omega_e\frac{\partial}{\partial x}\ln\frac{B_0}{n_0}-\frac{\partial^2u_0}{\partial x^2}\right)=0. \quad (31)$$

The condition (31) is a necessary but not sufficient condition for the in0stability to exist.

To avoid singularity in Eq. (28), the oscillations have to be in resonance with the flow exactly at the point where the necessary instability condition is satisfied, i.e.,

$$\mathrm{Re}\,\omega=k_yu_0(x_0), \quad \left.\left(\Omega_e\frac{\partial}{\partial x}\ln\frac{B_0}{n_0}-\frac{\partial^2u_0}{\partial x^2}\right)\right|_{x=x_0}=0. \quad (32)$$

To find the unstable oscillations, assume that the condition (32) is satisfied and that the unstable wave is in resonance with the flow, and then rewrite (28) as

$$\frac{\partial^2\phi}{\partial x^2}-k_y^2\phi+\frac{\Lambda(x)}{u_0(x_0)-u_0(x)}\phi=0, \quad (33)$$

where

$$\Lambda(x)=\left(\Omega_e\frac{\partial}{\partial x}\ln\frac{B_0}{n_0}-\frac{\partial^2u_0}{\partial x^2}\right).$$

Equation (33) in certain cases may have, according to the oscillation theorem,[12] a discrete set of eigenfunctions $\phi^{(n)}$, to which correspond eigenvalues $k^{(n)}$ and hence frequencies $\omega^{(n)}=k^{(n)}u_0(x_0)$, only if

$$U(x)=\frac{\Lambda(x)}{u_0(x_0)-u_0(x)}>0. \quad (34)$$

The exact criterion for the existence of the eigenfunction $\phi^{(n)}$ cannot be determined without specifying the exact form of the profile of $U(x)$. It is obvious at the same time that Eq. (33) has no eigenfunctions if $\Lambda(x)/[u_0(x_0)-u_0(x)]<0$, when the effective potential $U(x)$ forms not a "well" but a "hump." For example, if $U(x)\equiv0$, it is obvious that the resulting equation $\phi''-k_y^2\phi=0$ does not have a nonzero solution with the specified boundary conditions (30).

Condition (34) is in fact an estimate, sufficient to the existence of the unstable solution. Without specifying the exact form of axial distribution of parameters, it can be only stated that the effective potential "well" in (33), characterized by $U(x)$, should be deep and wide enough.

If we make an assumption that the set of eigenfunctions $\phi^{(n)}$ and eigenvalues $k^{(n)}$ exists, we can show that the oscillations with the values of k_y somewhat smaller than $k^{(n)}$ are unstable.

Let us consider oscillations with $k_y=k^{(n)}+\delta k$, $\omega=k^{(n)}u_0(x_0)+i\,\mathrm{Im}\,\omega$, where $\delta k\ll k^{(n)}$, and $\mathrm{Im}\,\omega\ll k^{(n)}u_0(x_0)$.

When we substitute these into (33), then we obtain

$$\mathrm{Im}\,\omega\propto\delta k\,U(x)|u_0'(x_0)|^2\int dx|\phi(x)|^2|\phi(x_0)|^{-2}. \quad (35)$$

Since we have assumed the requirement discussed above, that $U(x_0)>0$, then the oscillations will be unstable, i.e., $\mathrm{Im}\,\omega>0$ only for $\delta k<0$.

We must note that for the wave, which satisfies the Rayleigh necessary conditions (32), both the numerator and the denominator in (34) go to 0 at the resonant point $x=x_0$, therefore according to l'Hopitale rule

$$U(x_0)=\left.\frac{\Lambda(x)}{u_0(x_0)-u_0(x)}\right|_{x=x_0}=-\frac{\Lambda'(x_0)}{u_0(x_0)}. \quad (36)$$

It was already noted, that the presence of the point, where the inhomogeneity factor $\Lambda(x)$ goes to zero is not sufficient for the flow to be unstable. Taking the form of Eq. (33) into consideration, it is possible to assume that with the increase of $U(x)$ the first eigenfunction appears at a zero eigenvalue $k^{(1)}$ and further increase of $U(x)$ will increase the value of $k^{(1)}$. It was shown already that unstable oscillations should have $k<k^{(i)}$, therefore the appearance of the $k^{(1)}=0$ is the point of marginal stability of the flow in the slab geometry.

We should take into account that the case we consider is physically different from the classical problem now only by the more complicated form of the function $\Lambda(x)$ which now includes the gradients of plasma density and magnetic field. At the same time for the wave-like disturbance propagating azimuthally in the annular channel of the Hall thruster the possible wave number k_y is limited to the set

$$k_n = \frac{n}{R}, \quad n = 1,2,...,$$

where R is the radius of the acceleration channel. This means that the existence of the zero eigenvalue $k^{(1)}=0$ is not sufficient to allow an unstable azimuthal mode. The general instability condition on the function $U(x)$ therefore should be such that the first eigenvalue satisfies the following criterion:

$$k^{(1)} > k_n^2 = \frac{n^2}{R^2}. \tag{37}$$

Therefore, the instability condition (34) is modified accordingly to

$$U(x_0) > k_1^2 = \frac{1}{R^2}. \tag{38}$$

Thus, for any parameter distribution and corresponding function $U(x)$ there will be a finite number of unstable modes, limited by the corresponding value of $k^{(n)}$.

IV. KELVIN–HELMHOLTZ-TYPE INSTABILITY

Derivation of exact criterion for the appearance of the eigenfunction $\phi^{(1)}$ requires knowing the exact form of the function

$$\Lambda(x) = \left(\Omega_e \frac{\partial}{\partial x} \ln \frac{B_0}{n_0} - \frac{\partial^2 u_0}{\partial x^2} \right).$$

The profiles of plasma density and the electric field distribution inside the thruster channel are subject to numerous research efforts[8] and obtaining the exact solution is rather complicated both theoretically[9] and experimentally.[7,10]

We consider therefore the simplest distribution of parameters inside the thruster channel, allowing us to find the unstable mode, its frequency and growth rate.

Let us consider the step-like distribution of all parameters, where at the resonant point $x_0=0$ the drift velocity u_0 is changing its value from v_1 to v_2, while

$$A(x) = \left(\Omega_e \frac{\partial}{\partial x} \ln \frac{B_0}{n_0} \right)$$

is changing from A to $-A$ through $A=0$, so that the necessary instability condition is satisfied at $x=0$. In fluid dynamics the instability of flow with a step-like transverse profile of velocity is well known[14] and is customarily called Kelvin–Helmholtz instability. A simplified case of such instability without density and magnetic field gradients has been considered earlier.[15]

When the kinks in the drift velocity profile and the inhomogeneity factor are smeared out, the profiles become similar to ones existing in Hall thrusters, and the Kelvin–Helmholtz-type instability described below becomes a Rayleigh instability.

In the following[16] we now introduce the new function

$$\psi = \frac{\phi}{\omega - k_y u_0}$$

for which (33) takes the form

$$\frac{d}{dx}\left((\omega - k_y u_0)^2 \frac{d\psi}{dx} \right) - k_y^2 (\omega - k_y u_0)^2 \psi - k_y (\omega - k_y u_0) A(x) \psi = 0. \tag{39}$$

Now we solve (39) separately for $x>0$ and $x<0$, using for simplicity the symmetrical boundary conditions:

$$\psi(x=a) = \psi(x=-a) = 0.$$

Then for $x>0$ we rewrite (39) as

$$\frac{d^2\psi}{dx^2} - k^2 \left(1 + \frac{A}{\omega - k v_1} \right) \psi = 0, \tag{40}$$

therefore

$$\psi = C_1 \sinh\left[k \left(1 + \frac{A}{\omega - k v_1} \right)^{1/2} \right] (a+x). \tag{41}$$

Similarly, for $x<0$ the solution is

$$\psi = C_2 \sinh\left[k \left(1 - \frac{A}{\omega - k v_2} \right)^{1/2} \right] (a-x). \tag{42}$$

From the continuity of the solution we immediately obtain for the coefficients $C_1 = C_2$, which we will drop in further calculations, while integration of Eq. (39) around $x=0$ yields the following matching condition for ψ:

$$(\omega - k u_0(x))^2 \frac{d\psi}{dx} \bigg|_{-\epsilon}^{+\epsilon} = 0. \tag{43}$$

We now substitute our solution into (43),

$$(\omega - k v_1)^2 \left(1 + \frac{A}{\omega - k v_1} \right) \cosh\left[ka \left(1 + \frac{A}{\omega - k v_1} \right) \right] + (\omega - k v_2)^2 \left(1 - \frac{A}{\omega - k v_2} \right) \times \cosh\left[ka \left(1 + \frac{A}{\omega - k v_2} \right) \right] = 0. \tag{44}$$

This is in fact the dispersion relation for the azimuthally propagating unstable mode with

$$\omega = k \frac{v_1 + v_2}{2} \pm \frac{1}{2} \sqrt{-k^2 (v_2 - v_1)^2 - 2Ak(v_2 - v_1)}. \tag{45}$$

We have two modes here with the frequency $\omega = k[(v_1 + v_2)/2]$, one of which is unstable with the growth rate

$$\gamma = \tfrac{1}{2} \sqrt{k^2 (v_2 - v_1)^2 + 2Ak(v_2 - v_1)}. \tag{46}$$

When the velocity jump and the profile of the inhomogeneity parameter $A(x)$ are smeared out, the obtained solution represents a wave which is in phase $\omega = k_y (v_1 + v_2)/2$

with the electron flow at the center point where the parameter $\Lambda(x)$ turns into 0, thus satisfying the necessary condition for Rayleigh instability.

V. CONCLUSION

The theoretical model presented here predicts that an azimuthally propagating mode may become unstable if certain conditions on the axial distribution of parameters inside the thruster channel are met, most importantly when the parameter

$$\left(\Omega_e \frac{\partial}{\partial x} \ln \frac{B_0}{n_0} - \frac{\partial^2 u_0}{\partial x^2}\right)$$

is equal to 0 at some point inside the channel. This instability is predicted to be in resonance with the azimuthal electron flow at the same point the instability condition is satisfied. The derived instability condition applied to azimuthally symmetric geometry of Hall thruster shows that several modes with multiple frequencies can be unstable.

For a kilowatt-range Hall thruster (SPT-100 class or similar) typical values of the applied magnetic and electric fields are 200 G and 100 V/cm correspondingly. The electron drift velocities in this type of thrusters are therefore of the order of 10^6 m/s and the frequency of the unstable waves should be in the 1–10 MHz range. Given the characteristic plasma frequencies for such discharges (see Ref. 3) this validates the assumptions on the frequency of the wave, made during the derivation of the dispersion relation.

Further progress in the theoretical analysis of this instability is possible only after a detailed and accurate model of steady state processes inside Hall thruster channel is derived—a task beyond the scope of this paper.

Most importantly, experimental studies have indicated the presence of the purely azimuthal high-frequency ($f \sim 5-50$ MHz) waves inside the channel of the laboratory Hall thruster.[5] In certain operating regimes waves with multiple frequencies (harmonics) were detected.[5,6] Experimental studies of the steady-state distribution of plasma parameters[7] preliminarily suggest, that the detected high-frequency modes has the phase velocity the same or close to the electron drift velocity inside the channel for a wide range of thruster configurations and operating conditions. While there are other theoretically predicted unstable waves in this band, the experimental characteristics indicate that the experimentally observed wave is probably the Rayleigh-type instability discussed in this paper.

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