Anode sheath in Hall thrusters

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A set of hydrodynamic equations is used to describe quasineutral plasma in ionization and acceleration regions of a Hall thruster. The electron distribution function and Poisson equation are invoked for description of a near-anode region. Numerical solutions suggest that steady-state operation of a Hall thruster can be achieved at different anode sheath regimes. It is shown that the anode sheath depends on the thruster operating conditions, namely the discharge voltage and the mass flow rate. [DOI: 10.1063/1.1615307]

The Hall thruster (HT) is an electric propulsion device, in which axial electric and radial magnetic fields are applied in the coaxial channel [Fig. 1(a)]. The radial magnetic field impedes electron motion toward the anode. This provides sufficient electron density for effective ionization and allows the presence of a significant electric field in quasineutral plasma. The heavier ions, which are not affected by the magnetic field, are accelerated electrostatically toward the exit to produce thrust.

Various numerical models were proposed to describe physical processes in the HT.[1–6] In one-dimensional (1D) or quasi-1D (considering wall losses) modeling with a given temperature profile, one solves two ordinary differential equations for plasma density, $n(z)$, and ion flux, $J_i(z)=n(z)V_i(z)$, of the form:

$$dJ_i/dz=F(J_i,n,I_d),$$

$$dn/dz=G(J_i,n,I_d)/(1-V_i^2/V_s^2), \tag{1}$$

where F and G are nonlinear functions, V_i and V_s are the ion flow and ion acoustic velocities, respectively, and parameter I_d is the discharge current. A second-order system with one parameter requires three boundary conditions (BCs), or other constraints, whereas only the discharge voltage, V_d, is an experimentally given one. In dealing with these types of equations, Fruchtman *et al.*[3] proposed that requiring the solution to be regular at the sonic transition point, $V_i=V_s$, can be used as one of the constraints. The other was given by assuming zero ion flow at the anode, $z=0$. Rather than employing $V_i(0)=0$, Ahedo *et al.*[4] employed $V_i(0)=-V_s$, assuming the presence of the anode sheath and a back ion flow at the anode. This BC gives a physically valid solution over a wide range of discharge voltages. However, it was shown in Ref. 5 that for discharge voltages greater than a certain value this BC becomes inappropriate and another, "no sheath" type BC must be employed: $V_e(0)=-V_{\max}$, where V_e is the electron axial velocity and $V_{\max}$ is determined only by the electron distribution function (EDF) at the anode. Solutions obtained with this BC showed that an increase of a discharge voltage leads to the decrease of a back ion flow, and at a certain voltage: $V_i(0)=0$. The decrease of a back ion flow in the anode region of a HT was also discussed by Keidar *et al.*[2] In this letter, we present a nonquasineutral model, which describes the near-anode region in Hall thrusters, and show how an anode sheath and a back ion flow vary with the discharge voltage.

We use input parameters typical for the state of the art subkilowat HT with 9 cm outer channel wall diameter:[7] discharge voltage $V_d=150–300$ V, propellant (xenon) mass flow rate $\dot{m}=1.7–3.0$ mg/s, and radial magnetic field profile, $B_r(z)$, shown in Fig. 1(b). The channel width, H_{ch}, is 1.8 cm. Since the experimentally measured electron temperature is almost constant near the anode and the maximum temperature is of order of 20 eV,[8] similar to Ref. 5, we assume the

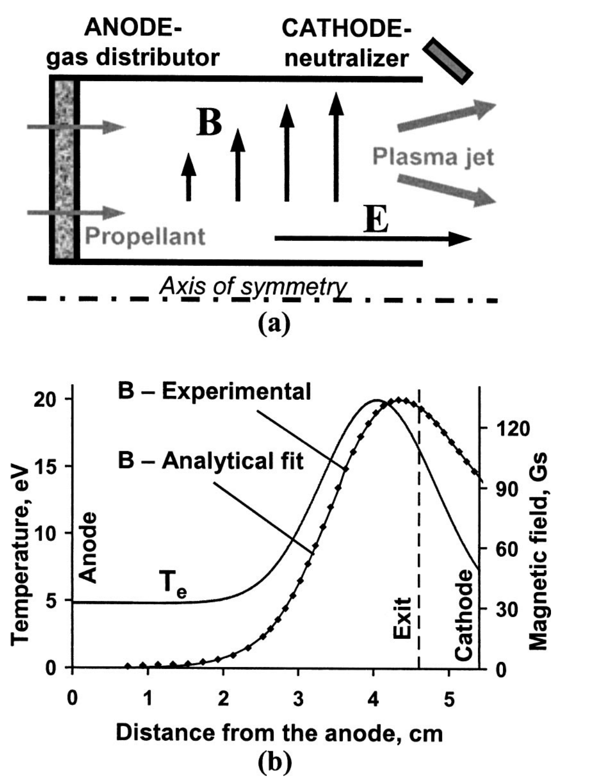

FIG. 1. (a) A schematic picture of a HT. (b) Radial magnetic field (B) and electron temperature (T_e) profiles.

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given electron temperature profile, $T_e(z)$, shown in Fig. 1(b). We also assume that plasma is quasineutral in a region with a strong magnetic field: $z>z_0$, where z is axial coordinate with $z=0$ at the anode, and that in the near-anode region, $z<z_0$, electrons can move freely toward the anode. For numerical simulations, we use $z_0=0.8\,\mathrm{cm}$, at which $\rho_{\mathrm{Be}}(z_0)/z_0\sim 10$, where ρ_{Be} is the electron gyroradius. This provides that electron trajectories are almost not affected by a magnetic field for $z<z_0$. The physical processes for $z>z_0$ can be expressed mathematically as follows:

$$J_i' = \langle\sigma V\rangle n_a n - 0.55n\sqrt{T_e/M_i}\times 2/H_{\mathrm{ch}}, \tag{2}$$

is the ion continuity equation. Here, the prime sign denotes the derivative with respect to z, M_i is the ion mass, T_e is the electron temperature, and n_a is the neutral density. The first term in Eq. (2) represents ionization and the second represents wall losses averaged over a channel width. Wall losses are considered zero outside the channel. The ionization coefficient, $\langle\sigma V\rangle(T_e)$, and the factor 0.55 are explained in Ref. 5.

$$(J_i V_i)' = eEn/M_i - 0.55n\sqrt{T_e/M_i}\times 2/H_{\mathrm{ch}}\times V_i + \langle\sigma V\rangle n_a n\times V_{a0}, \tag{3}$$

is the ion momentum equation. Here, E is the axial projection of the electric field. The first term on the right-hand side of Eq. (3) is the ion acceleration in the electric field. The second and the third terms originate from ion losses to the wall with the ion flow velocity, V_i, and their birth with the neutral flow velocity, V_a, respectively,[5]

$$n_a V_{a0} + J_i = J_m, \tag{4}$$

represents mass conservation. Here, $J_m=\dot{m}/(M_i A_{\mathrm{ch}})$ is the propellant flux, where A_{ch} is the channel cross section

$$-en\mu_e^{-1}V_e = eEn + (n_e T_e)', \tag{5}$$

is the phenomenological electron momentum equation, in which μ_e is the absolute value of the electron axial mobility in a radial magnetic field. We assume a modified Bohm diffusion with $\mu_e=\alpha/[16B_r(z)]$, where $\alpha=(1/8-1/6)$ is a fitting parameter[5,6]

$$-nV_e + J_i = J_d, \tag{6}$$

represents charge conservation. Here, $J_d=I_d/(eA_{\mathrm{ch}})$, where e is electron charge.

As was mentioned above, system (2)–(6) can be reduced to system (1), for which a solution can be constructed if three parameters are specified at z_0: n_0, V_{i0}, and V_{e0}. It can be shown that for each pair of velocities, there is only one value of n_0, which allows a solution to be regular at the sonic transition point: $V_i=V_s$.[5] To determine V_{i0} and V_{e0}, we need to consider the near-anode region: $0<z<z_0$. We again use Eqs. (2)–(4) with $n\rightarrow n_e$ in ionization terms and $n\rightarrow n_i$ in wall losses terms to describe the ion dynamics. However, unlike Refs. 3–5, we determine the electric field by using Poisson's equation:

$$d^2\varphi/dz^2 = 4\pi e(n_e - n_i), \tag{7}$$

rather than Eqs. (5) and (6). In Eq. (7), φ is the electric potential and $n_{e,i}$ are the electron and ion densities, respectively. In order to find the electron density, we solve 1D Vlasov's kinetic equation for electron velocity distribution function with the following assumptions: (a) EDF $(z=0)$ is half-Maxwellian with $T_0=T_e(z_0)$, i.e., there is no electron

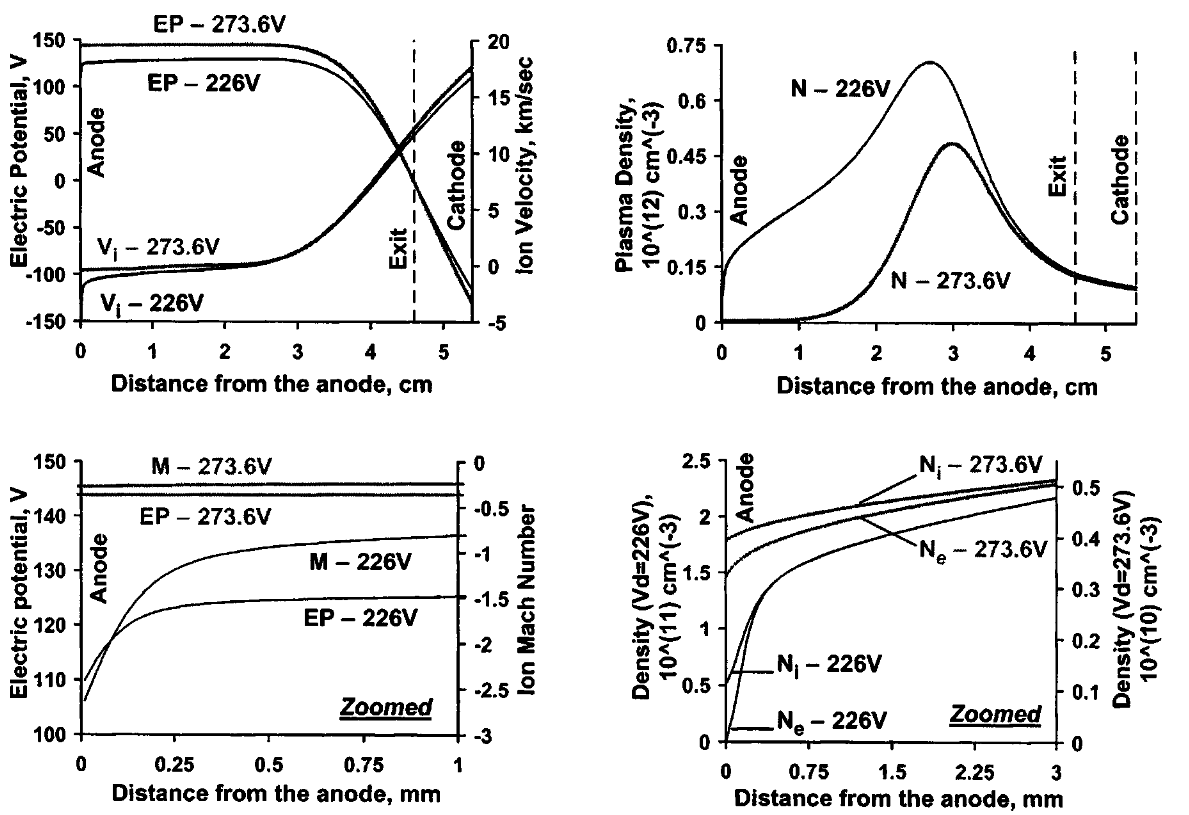

FIG. 2. The numerically obtained electric potential, (EP), ion velocity (V_i), ion Mach number (M), plasma density (N), electron density (N_e), and ion density (N_i) profiles for $\dot{m}=1.7$ mg/s and two discharge voltages: $V_d=226$ V and $V_d=273.6$ V. The bottom charts are focused on the near-anode region.

flux out of the anode, (b) the symmetric part of EDF ($z>0$) is Maxwellian with $Te=T_0$, (c) $B_r(z)=0$, and (d) $n_e(z_0)=n_0$. As can be seen from Fig. 1(b), the radial magnetic field is indeed negligible and $T_e=$constant for $z<z_0$. Integration of the EDF over all velocities gives:

$$n_e=n_0V_{e0}/\sqrt{2T_0/\pi m_e}\times\exp(e(\varphi-\varphi_a)/T_0)\times(1+\mathrm{erf}(\sqrt{e(\varphi-\varphi_a)/T_0})), \quad (8)$$

$$\exp(e(\varphi_a-\varphi_0)/T_0)/(1+\mathrm{erf}(\sqrt{e(\varphi_0-\varphi_a)/T_0}))=V_{e0}/\sqrt{2T_0/\pi m_e}, \quad (9)$$

where $\varphi_a=\varphi(0)$ is the anode potential, and $\varphi_0=\varphi(z_0)$ can be chosen arbitrarily.

If V_{i0} and V_{e0} are specified, we can solve Eqs. (2)–(6) for $z>z_0$ to find $n_i(z_0)=n_0$ and $\varphi'(z_0)$, and then solve Eqs. (2)–(4) and (7)–(9) for $z<z_0$. We constructed numerical solutions for several discharge voltages and found that: (1) for each value of V_{e0}, there is only one value of V_{i0} that results in $\varphi(z)=\varphi_a$ at $z=0$, (2) for each value of V_d, there is only one value of V_{e0} that results in $\varphi_{\mathrm{anode}}-\varphi_{\mathrm{cathode}}=V_d$.

The numerically obtained profiles for $V_d=226$ V and 273.6 V and a mass flow rate of 1.7 mg/s are shown in Fig. 2. As can be seen, about 50% of the discharge voltage drops outside the channel, which is in an agreement with experiment,[9] as well as the obtained propellant utilization (80%–90%). It was corroborated that steady-state operation of a HT at moderate discharge voltages requires the presence of a negative anode sheath and a back ion flow. For example, for $V_d=226$ V ion Mach number reaches one at about 0.5 mm from the anode, indicating the establishment of the anode sheath. However, an increase of a discharge voltage leads to the decrease of a negative anode sheath and, consequently, makes it disappear. At large discharge voltages, the electron axial velocity in quasineutral plasma near the anode becomes on the order of the electron thermal velocity, rendering the sheath unnecessary. It was also shown that at the same discharge voltage, the anode sheath increases with the increase of a mass flow rate. Ionization becomes more effective at larger mass flow rates, while the discharge current in HTs is limited by a magnetic field profile, which results in the described behavior of the anode sheath.

The steady-state quasi-1D model of a HT presented in this work showed that at certain operating conditions, for example, large discharge voltages, thruster operation does not require the presence of a negative anode sheath. Similar to glow discharges,[10] it may be expected that the further increase of V_d may lead to the presence of a positive anode sheath. The above results were obtained for the thruster model with a given temperature profile. If the electron temperature and, therefore, the electron thermal velocity, near the anode increase with the discharge voltage, one can expect that larger discharge voltages may still require the presence of a negative anode sheath.

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